PERGAMON

International Journal of Solids and Structures 37 (2000) 5233–5245

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Interaction of elastic waves with the unilateral interface between a layer and a half-space

Bing-Zheng Gai*

Department of Astronautics and Mechanics, P.O.Box 344, Harbin Institute of Technology, Harbin 150001, People's Republic of China

Received 30 September 1998; in revised form 3 August 1999

Abstract

In this paper, the interaction of an incident harmonic plane elastic wave with the unilateral interface between a layer and a half-space has been investigated. Based on the general traveling solutions of the linear governing equations for the layer and half-space, a method to solve this kind of the problems by means of functional equations has been proposed. Under the conditions of neglecting friction between the layer and half-space and the incident angle less than the critical value, the concrete solution for the problem has been given. The results show that the nonlinear distortions of the elastic wave field in the layer and half-space have arisen, which depend only on two dimensionless parameters relative to the properties of media, wave number, applied pressure and thickness of the layer. Similar to the case for infinite media, when thickness of the layer is finite, the leading edge of the separate zone at the interface is smooth, the distribution of the compressive stress is continuous, but not smooth at the trailing edge and result here in the jump of the compressive stress. The extent of the separate zone increase with the decrease of thickness of the layer.

Keywords: Elastic waves; Unilateral interface; Half-space with a layer; Functional equation method

1. Introduction

In the classical elastic theory, it is extremely important that the influence of the demarcation face between media, i.e. interface, upon the elastic wave filed (Ewing, 1957; Achenbach, 1973). In the past research works, the preponderance of the work has been concerned with the bilateral interface, i.e. that across which the field quantities are continuous. But, a lot of interface in the practice problems has no bilateral characteristics, for example the contact face between the engineering structure members or the machine parts, the fault plane in the earth's crust, the crack face in materials and so on. They can't hold

* Corresponding author.

PII: S0020-7683(99)00221-8

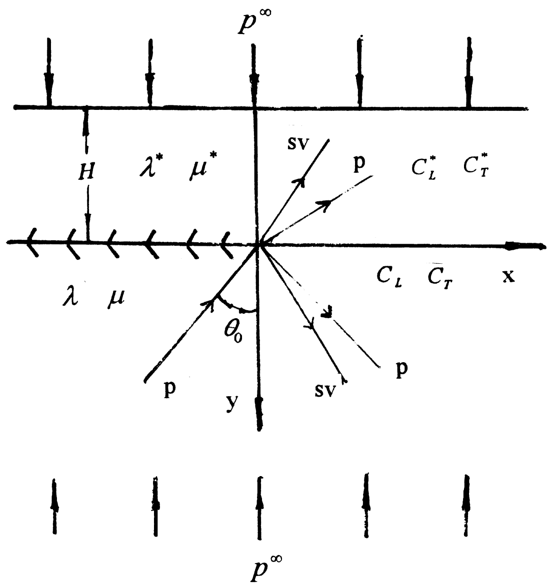

Fig. 1. Interaction of elastic waves with the unilateral interface between a layer and a half-space.

tensile traction, but only shear traction through friction. In contrast with bilateral interface, Comninou and Dundurs (1977) refer to this kind of interfaces as the unilateral interface. In the elastic wave field, the constrain of the unilateral interface on the field quantities is a kind of the nonlinear constrain, and problems corresponding to it are more complex than that for the bilateral interface with the linear constrain. Since 1977, the problems of the elastic wave propagation under the unilateral interface were independently deeply attacked by Miller (1977, 1978, 1979); Miller and Tran (1981a, 1981b) using the method of equivalent linearization, and by Comninou and Dundurs (1977, 1978a, 1978b, 1979a, 1979b, 1980); Comninou et al. (1982); Dundurs and Comninou (1979); Barber (1982); Chen et al. (1983) using the corrective method for the bilateral solutions, the important distortions behaviors in this kind of the elastic wave field, such as the increase in frequencies, phase shift, dissipation of energy etc., have been revealed. Recently, Wang et al. (1997, 1998) have intensively investigated the problems of re-polarizations for this kind of the elastic wave field, Zharii et al. (1995, 1996) discussed in detail theoretical modeling of a traveling wave ultrasonic motor related to it. However, it should be noted that the method of equivalent linearization is an approximate approach which make the nonlinear constrains at the interface linearization based on the certain assumptions, therefore, available to analyzing the anti-plane(or plane) problems in which media can be slided along the interface, but not separated (Sesawa and Kanai, 1940; Kanai, 1961; Iwan, 1973). As the corrective method for the bilateral solutions is an analytical method based on the bilateral solutions, consequently, it demands the bilateral solution for the problem to be solved is existent and relatively simple, otherwise, the solution for the problem is impossible or very difficult (Comninou and Dundurs, 1977, 1978a, 1978b). Precisely because of this, so far the solved problems have been principally confined to that in infinite media with relation to two half-space. In this paper, the interaction of an incident harmonic plane elastic wave with the unilateral interface between a layer with the finite thickness and an elastic half-space has been considered. Based on the general traveling solution of the linear governing equations for the layer and half space, a method to solve this kind of problems by means of functional equations has been proposed. Under the conditions of neglecting friction between the layer and half-space and the incident angle less than the critical value, the concrete solution for the problem has been given. The results show that the nonlinear distortions of the elastic wave field in the layer and half-space have arisen, which depend only on two dimensionless parameters relating to the properties of media, wave number, applied pressure and thickness of the layer. Similar to the case for infinite media, when thickness of the layer is finite, the leading edge of the separate zone at the interface is smooth, the distribution of the compressive stress is continuous; but not smooth at the trailing edge and result here in the jump of the compressive stress. The extent of the separate zone increase with the decrease of thickness of the layer.

2. Mathematical statement

The problem treated here is shown on Fig. 1. An elastic layer with thickness H is forced on an elastic half-space by the applied pressure p^{∞}. Suppose, the layer and half-space are the isotropic linear elastic media with Lame constants, longitudinal and transversal wave velocity λ, μ, c_{L}, c_{T} and λ^*, μ^*, c_{L}^*, c_{T}^*, respectively. There is no friction between the layer and half-space. A harmonic plane P or SV wave strikes the interface between them from the half-space at angle of incidence θ_0. Thus, the elastic wave fields in the layer and half-space are as follows

$$\varphi = f_{\varphi}(\zeta - p_{\varphi}y) + g_{\varphi}(\zeta + p_{\varphi}y) \tag{1a}$$

$$\psi = f_{\psi}(\zeta - p_{\psi}y) + g_{\psi}(\zeta + p_{\psi}y) \quad \text{(in the half-space)} \tag{1b}$$

$$\varphi^* = f_{\varphi}^*(\zeta - p_{\varphi}^*y) + g_{\varphi}^*(\zeta + p_{\varphi}^*y) \tag{1c}$$

$$\psi^* = f_{\psi}^*(\zeta - p_{\psi}^*y) + g_{\psi}^*(\zeta + p_{\psi}^*y) \quad \text{(in the layer)} \tag{1d}$$

whereφ, ψ, φ^*, ψ^* are Lame potential functions in the half-space and layer respectively; f_{φ}, g_{φ}, f_{ψ}, g_{ψ}, f_{φ}^*, g_{φ}^*, f_{ψ}^*, g_{ψ}^* are C^2 kind real function of their arguments, f_{φ}, f_{ψ} are known incident waves; $\zeta = x - ct$ is the moving coordinate at the interface $y = 0$, $c = c_0/\sin\theta_0$ is the apparent velocity of propagation of elastic waves along the interface; c_0 is incident wave velocity which equals c_{L} (P wave incidence)or c_{T} (SV wave incidence); $p_{\varphi} = \sqrt{(c/c_{\mathrm{L}})^2 - 1}$, $p_{\psi} = \sqrt{(c/c_{\mathrm{T}})^2 - 1}$, $p_{\varphi}^* = \sqrt{(c/c_{\mathrm{L}}^*)^2 - 1}$, $p_{\psi}^* = \sqrt{(c/c_{\mathrm{T}}^*)^2 - 1}$; x, y are space coordinates; t is time. Here, only the case considered is that the incident angle θ_0 does not exceed the critical value, i.e. $\theta_0 < \theta_{\mathrm{cr}} \equiv \sin^{-1}(c_{\mathrm{L}}/c_{\mathrm{L}}^*)$ (P wave incidence) or $\min(\sin^{-1}(c_{\mathrm{T}}/c_{\mathrm{L}}), \sin^{-1}(c_{\mathrm{T}}/c_{\mathrm{L}}^*))$, (SV wave incidence), thus, in this case p_{φ}, p_{ψ}, p_{φ}^*, p_{ψ}^* are all real numbers.

From Eqs. (1a)–(1d), the displacement and stress in the half-space and layer are easily obtained as follows

$$u = f_{\varphi}' + g_{\varphi}' - p_{\psi}\left(f_{\varphi}' - g_{\psi}'\right) \tag{2a}$$

$$v = p_{\varphi}\left(-f_{\varphi}' + g_{\varphi}'\right) - f_{\psi}' - g_{\psi}' \tag{2b}$$

$$\sigma_x = \left[A + 2\mu\left(1 - p_{\phi}^2\right)\right]\left(f_{\varphi}'' + g_{\varphi}''\right) - B\left(f_{\psi}'' - g_{\psi}''\right) \tag{2c}$$

$$\sigma_y = A\left(f_{\varphi}'' + g_{\varphi}''\right) + B\left(f_{\psi}'' - g_{\psi}''\right) \tag{2d}$$

$$\tau_{xy} = \mu\left(p_\psi^2 - 1\right)\left[R\left(-f_\varphi'' + g_\varphi''\right) + f_\psi'' + g_\psi''\right] \tag{2e}$$

$$u^* = f_\varphi^{*'} + g_\varphi^{*'} - p_\psi^*\left(f_\psi^{*'} - g_\psi^{*'}\right) \tag{3a}$$

$$v^* = p_\varphi^*\left(-f_\varphi^{*'} + g_\varphi^{*'}\right) - f_\psi^{*'} - g_\psi^{*'} \tag{3b}$$

$$\sigma_x^* = \left[A^* + 2\mu\left(1 - p_\phi^2\right)\right]\left(f_\varphi^{*''} + g_\varphi^{*''}\right) - B^*\left(f_\psi^{*''} - g_\psi^{*''}\right) \tag{3c}$$

$$\sigma_y^* = A^*\left(f_\varphi^{*''} + g_\varphi^{*''}\right) + B^*\left(f_\psi^{*''} - g_\psi^{*''}\right) \tag{3d}$$

$$\tau_{xy}^* = \mu^*\left(p_\psi^{*^2} - 1\right)\left[R^*\left(-f_\varphi^{*''} + g_\varphi^{*''}\right) + f_\psi^{*''} + g_\psi^{*''}\right] \tag{3e}$$

where u, v, σ_x, σ_y, τ_{xy} and u^*, v^*, σ_x^*, σ_y^*, τ_{xy}^* are displacements in x, y directions, normal stresses and shear stresses in the half-space and layer, respectively; $A = \lambda + (\lambda + 2\mu)p_\varphi^2$, $B = 2\mu p_\psi$, $R = 2p_\varphi/\sqrt{p_\psi^2 - 1}$; $A^* = \lambda^* + (\lambda^* + 2\mu^*)p_\varphi^{*2}$, $B^* = 2\mu^* p_\psi^*$, $R^* = 2p_\varphi^*/\sqrt{p_\psi^{*2} - 1}$; prime $'$ denotes the derivative of function with respect to its argument.

At the interface $y = 0$ between the layer and half-space, two kinds of zones can be usually arisen. One is that referred to as separate zone and denoted by I_{sz}; the other contact zone denoted by I_{cz}. In the two kinds of zones, the stress (the same as the bilateral interface) remain to be continuous, and the shear stress at the interface must be zero due to neglecting friction, thus, the conditions at the interface can be written as

$$\left.\begin{aligned} & v(\zeta) = v^*(\zeta), \qquad \sigma_y(\zeta) = \sigma_y^*(\zeta) \\ & \tau_{xy}(\zeta) = \tau_{xy}^*(\zeta) = 0 \\ & N(\zeta) = \sigma_y(\zeta) = \sigma_y^*(\zeta) \leqslant 0 \end{aligned}\right\} (\zeta \in I_{cz}) \tag{4}$$

$$\left.\begin{aligned} & \sigma_y(\zeta) = \sigma_y^*(\zeta) = 0 \\ & \tau_{xy}(\zeta) = \tau_{xy}^*(\zeta) = 0 \\ & \Delta v = v(\zeta) - v^*(\zeta) \geq 0 \end{aligned}\right\} (\zeta \in I_{sz}) \tag{5}$$

where $N(\zeta)$ is the normal traction of contact zone at the interface, $\Delta v(\zeta)$ is the gap of separate zone at the interface. The boundary conditions at the top surface $y = -H$ of the layer are

$$\sigma_y^*(\zeta) = -p^\infty, \tag{6a}$$

$$\tau_{xy}^*(\zeta) = 0 \tag{6b}$$

3. Solution of the problem

Substituting Eqs. (2a)–(2e), (3a)–(3e) into Eqs. (4), (5), (6a), (6b), we have in the contact zone $\zeta \in I_{cz}$

$$g''_{\varphi}(\zeta) = f''_{\varphi}(\zeta) - \frac{p^*_{\varphi} + R^*}{R^*(p_{\varphi} + R)}\left[f^{*''}_{\psi}(\zeta) + g^{*''}_{\psi}(\zeta)\right] \tag{7a}$$

$$g''_{\psi}(\zeta) = -f''_{\psi}(\zeta) + \frac{R}{R^*}\frac{(p^*_{\varphi} + R^*)}{(p_{\varphi} + R)}\left[f^{*''}_{\psi}(\zeta) + g^{*''}_{\psi}(\zeta)\right] \tag{7b}$$

$$\begin{aligned} f^{*''}_{\varphi}(\zeta) = {} & \frac{A}{A^*}f''_{\varphi}(\zeta) + \frac{B}{A^*}f''_{\psi}(\zeta) - \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{2A^*R^*(p_{\varphi} + R)} + \frac{A^* + B^*R^*}{2A^*R^*} - \frac{1}{R^*}\right] \times f^{*''}_{\varphi}(\zeta) \\ & - \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{2A^*R^*(p_{\varphi} + R)} + \frac{A^* - B^*R^*}{2A^*R^*} - \frac{1}{R^*}\right]g^{*''}(\zeta) \end{aligned} \tag{7c}$$

$$\begin{aligned} g^{*''}_{\varphi}(\zeta) = {} & \frac{A}{A^*}f''_{\varphi}(\zeta) + \frac{B}{A^*}f''_{\psi}(\zeta) - \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{2A^*R^*(p_{\varphi} + R)} + \frac{A^* + B^*R^*}{2A^*R^*}\right]f^{*''}_{\psi}(\zeta) \\ & - \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{2A^*R^*(p_{\varphi} + R)} + \frac{A^* - B^*R^*}{2A^*R^*}\right]g^{*''}_{\psi}(\zeta) \end{aligned} \tag{7d}$$

$$\begin{aligned} & \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{(A^* + B^*R^*)(p_{\varphi} + R)} - \frac{A^* - B^*R^*}{A^* + B^*R^*}\right]f^{*''}_{\psi}(\zeta) + \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{(A^* + B^*R^*)(p_{\varphi} + R)} - 1\right]g^{*''}_{\psi}(\zeta) \\ & + \frac{A^* - B^*R^*}{A^* + B^*R^*}f^{*''}_{\psi}\left[\zeta - (p^*_{\varphi} - p^*_{\psi})H\right] + g^{*''}_{\psi}\left[\zeta - (p^*_{\varphi} + p^*_{\psi})H\right] \\ & = \frac{2AR^*}{A^* + B^*R^*}f^{*''}_{\varphi}(\zeta) + \frac{2BR^*}{A^* + B^*R^*}f''_{\psi}(\zeta) + \frac{R^*p^{\infty}}{A^* + B^*R^*} \end{aligned} \tag{8a}$$

$$\begin{aligned} & \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{(A^* + B^*R^*)(p_{\varphi} + R)} + 1\right]f^{*''}_{\psi}(\zeta) + \left[\frac{(A+BR)(p^*_{\varphi} + R^*)}{(A^* + B^*R^*)(p_{\varphi} + R)} + \frac{A^* - B^*R^*}{A^* + B^*R^*}\right]g^{*''}_{\psi}(\zeta) \\ & - f^{*''}_{\psi}\left[\zeta + (p^*_{\varphi} + p^*_{\psi})H\right] - \frac{A^* - B^*R^*}{A^* + B^*R^*}g^{*''}_{\psi}\left[\zeta + (p^*_{\varphi} - p^*_{\psi})H\right] \\ & = \frac{2AR^*}{A^* + B^*R^*}f''_{\varphi}(\zeta) + \frac{2BR^*}{A^* + B^*R^*}f''_{\psi}(\psi) + \frac{R^*p^{\infty}}{A^* + B^*R^*} \end{aligned} \tag{8b}$$

in the separate zone, $\zeta \in I_{sz}$

$$g''_{\varphi}(\zeta) = -\frac{A - BR}{A + BR} f''_{\varphi}(\zeta) - \frac{2B}{A + BR} f''_{\psi}(\zeta) \tag{9a}$$

$$g''_{\varphi}(\zeta) = \frac{2AR}{A + BR} f''_{\varphi}(\zeta) - \frac{A - BR}{A + BR} f''_{\psi}(\zeta) \tag{9b}$$

$$f^{*''}_{\varphi}(\zeta) = \frac{A^* - B^*R^*}{2A^*R^*} f^{*''}_{\psi}(\zeta) + \frac{A^* + B^*R^*}{2A^*R^*} g^{*''}_{\psi}(\zeta) \tag{9c}$$

$$g^{*''}_{\varphi}(\zeta) = -\frac{A^* + B^*R^*}{2A^*R^*} f^{*''}_{\psi}(\zeta) - \frac{A^* - B^*R^*}{2A^*R^*} g^{*''}_{\psi}(\zeta) \tag{9d}$$

$$f^{*''}_{\psi}\left[\zeta + \left(p^*_{\varphi} + p^*_{\psi}\right)H\right] - f^{*''}_{\psi}(\zeta) + \frac{A^* - B^*R^*}{A^* + B^*R^*} \times \left\{g^{*''}_{\psi}\left[\zeta + \left(p^*_{\varphi} - p^*_{\psi}\right)H\right] - g^*_{\psi}(\zeta)\right\} = -\frac{R^*p^{\infty}}{A^* + B^*R^*} \tag{10a}$$

$$\frac{A^* - B^*R^*}{A^* + B^*R^*}\left\{f^{*''}_{\psi}\left[\zeta - \left(p^*_{\varphi} - p^*_{\psi}\right)H\right] - f^{*''}_{\psi}(\zeta)\right\} + g^{*''}_{\psi}\left[\zeta - \left(p^*_{\varphi} + p^*_{\psi}\right)H\right] - g^{*''}_{\psi}(\zeta) = \frac{R^*p^{\infty}}{A^* + B^*R^*} \tag{10b}$$

The incident harmonic plane P (or SV) wave may be taken as

$$f_{\varphi}(\zeta, y)\left(or\, f_{\psi}(\zeta, y)\right) = \mathrm{Re}F_0 e^{ik\left(\zeta - \frac{\cos\theta_0}{\sin\theta_0} y\right)} \tag{11}$$

where F_0 is a given complex number which prescribe the amplitude and initial phase of the incident wave and is taken here as $F_0 = |F_0|e^{i\frac{\pi}{2}}$; $k = k_0 \sin\theta_0$ is the apparent velocity for incident wave along interface; ω is the angular frequency for incident wave; *Re* express real part; $\cos\theta_0/\sin\theta_0 = p_{\varphi}$, $f_{\psi}(\zeta, y) = 0$ (P wave incidence) or $\cos\theta_0/\sin\theta_0 = p_{\psi}$, $f_{\phi}(\zeta, y) = 0$ (SV wave incidence). For the sake of simplicity, only the case considered below will be that for P wave incidence. In this case, according to the method constructing the solution for the linear functional equation (Kuczma, 1968; Peluch and Salkovskii, 1974), the solutions of the functional Eqs. (8a) and (8b) should be taken as

$$f^{*''}_{\psi}(\zeta) = \mathrm{Re}\left(C_f e^{ik\zeta}\right) + C^0_f \tag{12a}$$

$$g^{*''}_{\psi}(\zeta) = \mathrm{Re}\left(C_g e^{ik\zeta}\right) + C^0_g \tag{12b}$$

in which C_f, C_g, C^0_f, C^0_g are constants to be determined. Substituting Eqs. (12a) and (12b) into Eqs. (8a) and (8b) we have,

$$-C_f/k = C_g/k = A_{\psi} = \frac{(\delta_1 + i\delta_2)mAF_0}{\Delta_1 + i\Delta_2} \tag{13a}$$

$$C^0_f = C^0_g = A^0_{\psi} = \frac{1}{2}\frac{m}{l+1} p^{\infty} \tag{13b}$$

$$\Delta_1 = 1 - n^2 + n^2\cos\left[k\left(p_\varphi^* - p_\psi^*\right)H\right] - \cos\left[k\left(p_\varphi^* + p_\psi^*\right)H\right] \tag{13c}$$

$$\Delta_2 = (l+1)\left\{\sin\left[k\left(p_\varphi^* + p_\psi^*\right)H\right] - n \ast\ast\sin\left[k\left(p_\kappa^* - p_\psi^*\right)H\right\}\right. \tag{13d}$$

$$\delta_1 = (n+1) - n\cos\left[k\left(p_\varphi^* - p_\psi^*\right)H\right] - \cos\left[k\left(p_\varphi^* + p_\psi^*\right)H\right] \tag{13e}$$

$$\delta_2 = \sin\left[k\left(p_\varphi^* - p_\psi^*\right)H\right] - n\sin\left[k\left(p_\varphi^* - p_\psi^*\right)H\right] \tag{13f}$$

$$l = \frac{(A+BR)\left(p_\varphi^* + R^*\right)}{(A^* + B^*R^*)\left(p_\varphi + R\right)} - 1, \tag{13g}$$

$$m = \frac{R^*}{A^* + B^*R^*}, \tag{13h}$$

$$n = \frac{A^* - B^*R^*}{A^* + B^*R^*} \tag{13k}$$

Similarly, the solutions of the functional Eqs. (10a) and (10b) should be taken as

$$f_\psi^{*//}(\zeta) = S_f\zeta + S_f^0, \tag{14a}$$

$$g_\psi^{*//}(\zeta) = S_g\zeta + S_g^0 \tag{14b}$$

in which S_f, S_g, S_f^0, S_g^0 are constants to be determined. Substituting Eqs. (14a)–(14c) into Eqs. (10a), (10b) we have

$$S_f = S_g = \frac{m}{\left[(n+1)p_\phi^* - (n-1)p_\varphi^*\right]H} \tag{14c}$$

$$S_f^0 = S_g^0 = A_\varphi^0 = \frac{lm}{2(l+1)}p^\infty \tag{14d}$$

Thus, Eqs. (14a)–(14e) become

$$f_\psi^{*''}(\zeta) = g_\psi^{*''}(\zeta) = \frac{m}{2(l+1)}\left[1 - \frac{2(l+1)}{(n+1)p_\varphi^* - (n-1)p_\psi^*}\cdot\frac{\zeta}{H}\right]p^\infty \tag{14e}$$

which will coincide with Eqs. (12a), (12b), when no incident wave strikes the interface because, in this case, the whole interface will be closed under the pre-pressure p^∞ and the separate zone will vanish.

4. Interface State

At the contact zone, $\zeta \in I_{cz}$, from Eqs. (3a)–(3e), (7a)–(7d), (12a), (12b), we have

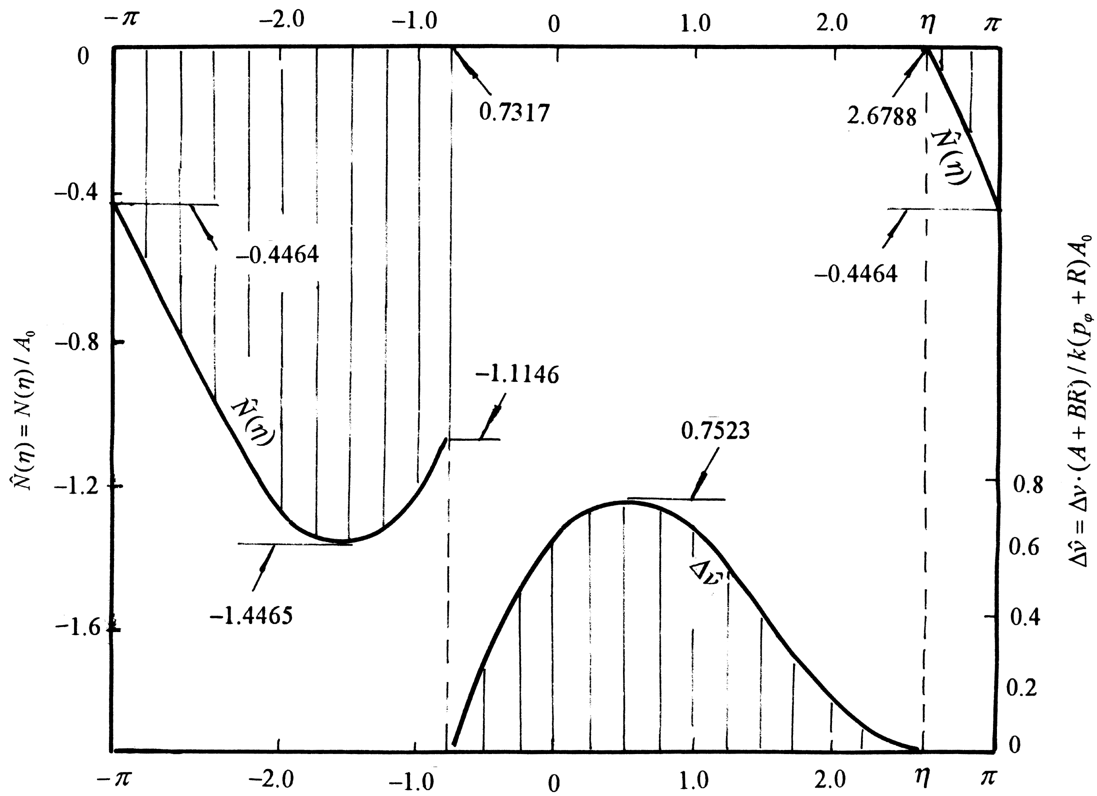

Fig. 2. Dimensionless traction $\hat{N}(\eta)$ and gap $\Delta\hat{v}$ at interface ($p^\infty/A_0=0.5$, $q/H_0=0.02$).

$$N(\eta)=N(\zeta)=2Af''_\varphi(\zeta)-\frac{l+1}{m}\left[f^{*''}_\psi(\zeta)+g^{*''}_\psi(\zeta)\right]=A_0\left(\sin\eta-\frac{p^\infty}{A_0}\right) \tag{15}$$

where $\eta=k\zeta$ is the dimensionless moving coordinate at the interface; $A_0=2k^2A|F_0|$. At the separate zone, $\zeta\in I_{sz}$ from Eqs. (2a)–(2e), (3a)–(3e), (9a)–(9d), (14a)–(14e) we have

$$\begin{aligned}k\Delta v'(\eta)=\Delta v'(\zeta)&=\frac{p^*_\varphi+R^*}{R^*}\left[f^{*''}_\varphi(\zeta)+g^{*''}_\psi(\zeta)\right]-2A\frac{p_\varphi+R}{A+BR}f''_\varphi(\zeta)\\&=\frac{A_0(p_\varphi+R)}{A+BR}\left[-\sin\eta+\frac{p^\infty}{A_0}\left(1-2\frac{q}{H_0}\eta\right)\right]\end{aligned} \tag{16}$$

where$q=(l+1)/[(n+1)p^*_\varphi-(n-1)p^*_\psi]$; $H_0=kH$ is dimensionless thickness of the layer.

Integrating the above equation, we have

$$\Delta v(\eta)=\frac{A_0(p_\varphi+R)}{k(A+BR)}\left[\cos\eta+\frac{p^\infty}{A_0}\eta\left(1-\frac{q}{H_0}\eta\right)-L\right] \tag{17}$$

where L is a constant. At the leading and trailing edges $\eta=\beta$ and α of the separate zone, since $\Delta v(\eta)=0$, thus, from the above equation, we have

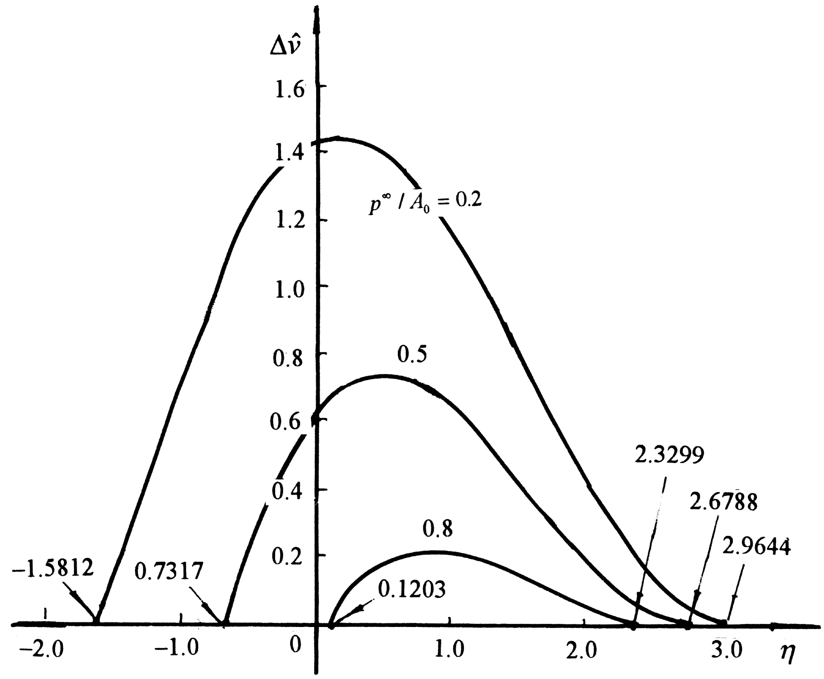

Fig. 3. Variations of interface gap $\Delta\hat{v}$ with η and p^{∞}/A_0 ($q/H_0 = 0.02$).

$$L = \frac{p^{\infty}}{A_0}\alpha\left(1 - \frac{q}{H_0}\alpha\right) + \cos\alpha = \frac{p^{\infty}}{A_0}\beta\left(1 - \frac{q}{H_0}\beta\right) + \cos\beta \tag{18}$$

At the same time, when a closed point at the interface enter into the separate zone, the stress in this point should be released out, hence, we also have

$$\sigma_y = \sigma_y^{*} = \frac{k(A + BR)}{p_{\varphi} + R}\Delta v'(\beta) = 0 \quad \left(\beta \geq \frac{\pi}{2}\right) \tag{19}$$

Substituting (16) into the above equation, we obtain

$$\sin\beta = \frac{p^{\infty}}{A_0}\left(1 - 2\frac{q}{H_0}\beta\right) \tag{20}$$

Eqs. (15), (17), (18) and (20) have fixed the distribution for the normal traction $N(\eta)$, gap $\Delta v(\eta)$ and the extent of the contact and separate zones. Only the interval $-\pi \leq \eta \leq \pi$ need be considered at the interface, because of the periodicity of all field quantities. Take dimensionless quantities

$$\hat{N}(\eta) = \frac{N(\eta)}{A_0}, \tag{21a}$$

$$\Delta\hat{v}(\eta) = \frac{k(A + BR)}{A_0(p_{\varphi} + R)}\Delta v(\eta) \tag{21b}$$

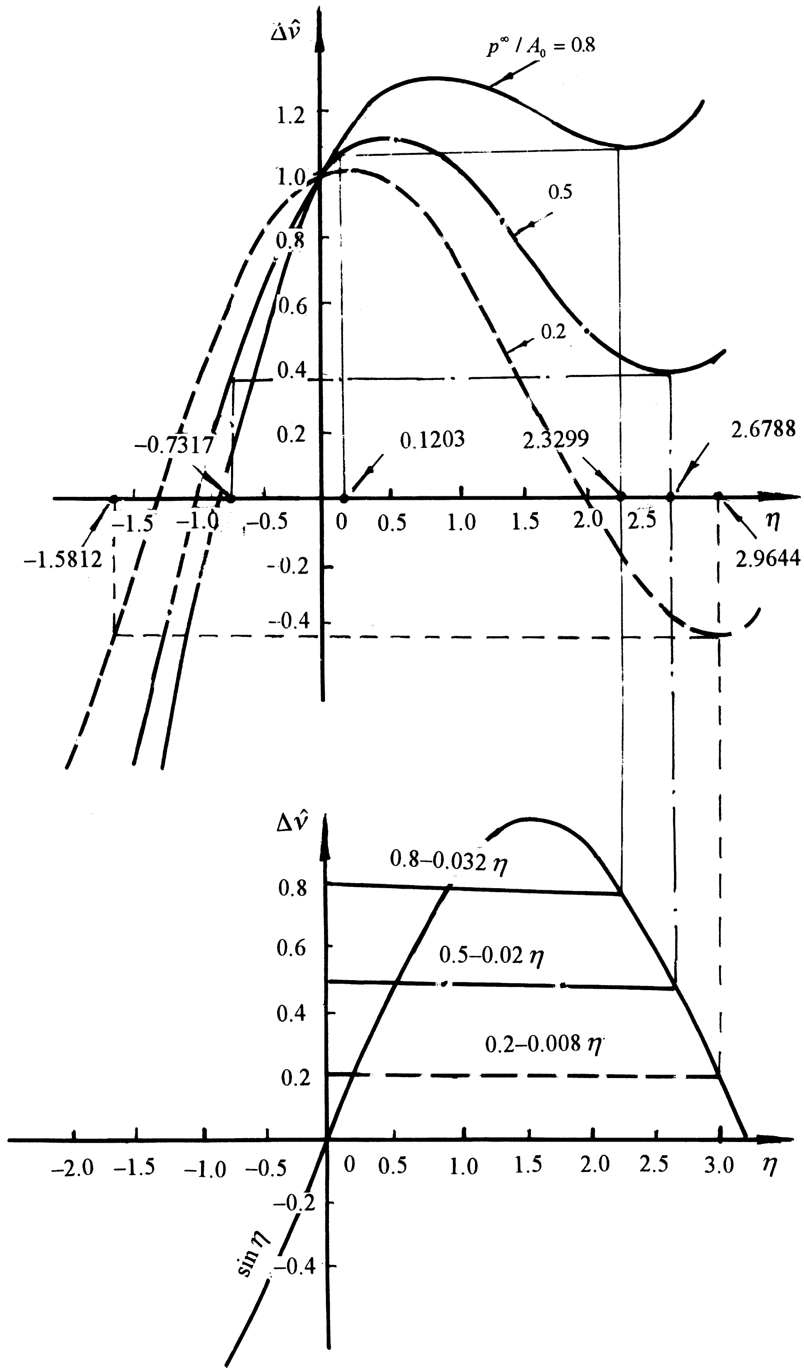

Fig. 4. Process determining separate zone at the interface ($q/H_0 = 0.02$).

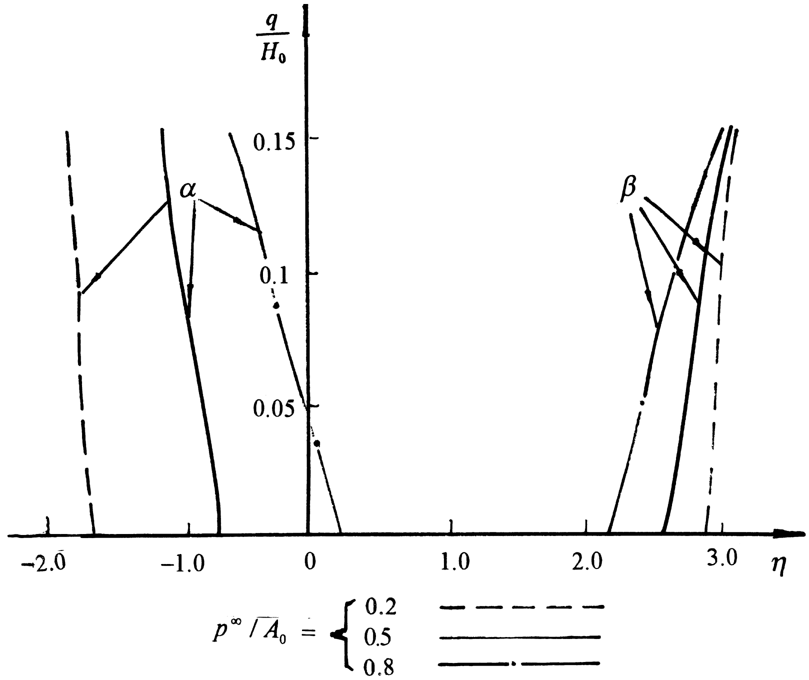

Fig. 5. Variations of extent of separate zone with q/H_0.

Fig. 2 shows the variation of $\hat{N}(\eta)$ and $\Delta\hat{v}(\eta)$ with η for the combined parameters $p^{\infty}/H_0 = 0.5$ and $q/H_0 = 0.02$. On the figure, the distribution curve of the dimensionless gap $\Delta\hat{v}(\eta)$ is a hill in shape with vanishing slope at the leading edge ($\eta = 2.6788$), and is oblique at the trailing edge ($\eta = -0.7317$), where the jump with amplitude -1.1146 appears for the distribution curve of the dimensionless normal traction $\hat{N}(\eta)$. Fig. 3 shows the variations of $\Delta\hat{v}(\eta)$ with η for the combined parameters $q/H_0 = 0.02$ and $p^{\infty}/A_0 = 0.2$, 0.5, 0.8. We can see from this figure that the amplitude and extent of gaps are all maximum for $p^{\infty}/A_0 = 0.2$, second for 0.5 and minimum for 0.8. Fig. 4 shows the process determining the separate zone by means of solving simultaneously, Eqs. (18) and (20). The curves on the figure are the calculating results when the values of the combined parameters are taken as $p^{\infty}/A_0 = 0.2$, 0.5, 0.8 and $q/H_0 = 0.02$. Fig. 5 shows the variations of the leading and trailing edge locations β, α, and the extent $(\beta - \alpha)$ of the separate zone with the combined parameters q/H_0 and p^{∞}/A_0 in which the solid line corresponds to $p^{\infty}/A_0 = 0.5$, the broken line to 0.2, the dot dash line to 0.8.

From the above analysis and the numerical calculating results, the following conclusions may be reached.

1. The interface state depends only on two combined parameters p^{∞}/A_0 and q/H_0. The former is a pre-pressure non-dimensioned by A_0 and independent of thickness of the layer; the latter is a reciprocal of thickness of the layer non-dimensional by k/q and independent of the pre-pressure, but both are relative to the properties of media, characteristics of the incident wave.
2. The separate zone at the interface will arise only when the condition

$$1 - \frac{1}{p^{\infty}/A_0} \leq \frac{q}{H_0} \tag{22}$$

is satisfied. When dimensionless thickness $H_0 \rightarrow \infty$, the above inequality becomes $(p^\infty/A_0) \leq 1$, which is the same with the results given by Comninou and Dundurs (1977) for the case of infinite media.

3. It is always smooth at the leading edge of the separate zone for any thickness of the layer. Consequently, here the traction at the interface remains continuous, but, generally, no smooth at the trailing edge, a jump of the traction at the interface will arise here, in other words, a crash between the layer and half-space will occur here in the elastic wave field.
4. The extent of the separate zone will decrease with the increase of parameter p^∞/A_0, and increase with the decrease of parameter q/H_0. When thickness of the layer $H_0 \rightarrow \infty$, pre-pressure $p^\infty \rightarrow 0$, i.e. for two half-space that does not apply pre-pressure, the unilateral interface between them will become a free surface from stress, which is the result given by Comninou and Dundurs (1977) for two half-space.

Acknowledgements

Support by the China National Science Foundation under Grant No. 19772010 is gratefully acknowledged.

References

Achenbach, J.D., 1973. Wave Propagation in Elastic Solids. North Holland, Amsterdam.

Barber, J.R., Comninou, M., Dundur, J., 1982. Contact transmission of wave motion between two solids with an initial gap. International Journal of Solids and Structures 18, 775–781.

Chez, E.L., Dundurs, J., Comninou, M., 1983. Energy relations for SH waves interacting with a frictional contact interface. International Journal of Solids and Structures 19, 579–586.

Sesawa, K., Kanai, K., 1940. A fault surface or block absorbs seismic waves energy Bulletin Earthquake Research Institute, Tokyo University, vol. 18, pp. 465–482.

Comninou, M., Dundurs, J., 1977. Reflection and refraction of elastic waves in presence of separation. Proceedings of the Royal Society of London Series A 356, 509–528.

Comninou, M., Dundurs, J., 1978a. Singular reflection and refraction of elastic waves due to separation. Journal of Applied Mechanics 45, 548–552.

Comninou, M., Dundurs, J., 1978b. Elastic interface waves and sliding between two solids. Journal of Applied Mechanics 45, 325–330.

Comninou, M., Dundurs, J., 1979a. Interaction of elastic waves with a unilateral interface. Proceeding of the Royal Society of London Series A 368, 141–154.

Comninou, M., Dundurs, J., 1979b. Interface separation caused by a plane elastic wave of arbitrary form. Wave Motion 1, 17–23.

Comninou, M., Dundurs, J., 1980. Interface slip caused by an SH pulse. International Journal of solids and Structures 16, 283–289.

Comninou, M., Barber, J.R., Dundurs, J., 1982. Disturbance at a frictional interface caused by a plane elastic pulse. Journal of Applied Mechanics 49, 361–365.

Dundurs, J., Comninou, M., 1979. Interface separation in the transonic range caused by a plane stress pulse. Journal of Sound Vibration 62, 317–325.

Ewing, W.M., 1957. Elastic Wave in Layered Media. McGraw-Hill, New York.

Iwan, W.D., 1973. A generalization of the concept of equivalent linearization. International Journal of Non-Linear Mechanics 8, 279–287.

Kanai, K., A new problem concerning surface waves Bulletin Earthquake Research Institute, Tokyo University, vol. 39, 359–366.

Kuczma, M., 1968. Functional Equations in a Single Variable, Warszama.

Miller, R.K., 1977. An approximate method of analysis of the transmission of elastic waves through a friction boundary. Journal of Applied Mechanics 44, 652–656.

Miller, R.K., 1978. The effects of boundary friction on the propagation of elastic waves. Bulletin of the Seismology Society of America 68, 987–998.

Miller, R.K., 1979. An estimate of the properties of Love-type surface waves in a frictionally bonded layer. Bulletin of the Seismology Society of America 69, 305–317.

Miller, R.K., Tran, H.T., 1981a. Reflection, refraction and absorption of elastic waves at a frictional interface: SH motion. Journal of Applied Mechanics 46, 625–630.

Miller, R.K., Tran, H.T., 1981b. Reflection, refraction and absorption of elastic waves at a frictional interface: P and SV motion. Journal of Applied Mechanics 48, 155–160.

Peluch, G.P., Salkovskii, A.N., 1974. Introduction to Theory of Functional Equation. Nauk Dumka, Moscow in Russian.

Wang, Y.S., Yu, G.L., Gai, B.Z., 1997. Slip with friction between an elastic layer and a substrate caused by an SH pulse. Mechanic Research Communications 24, 85–91.

Wang, Y.S., Yu, G.L., Gai, B.Z., 1998. Re-polarization of elastic waves at a frictional contact interface. Part I: Incidence of an SH wave. International Journal of solids and structures 35, 2001–20021.

Zharii, O.Y., 1995. Adhesive contact between the surface wave and a rigid strip. Journal of Applied Mechanics 62, 368–372.

Zharii, O.Y., 1996. Frictional contact between the surface wave and a rigid strip. Journal of Applied Mechanics 63, 15–20.

Zharii, O.Y., Ulitko, A.F., 1995. Smooth contact between the running Rayleigh wave and rigid strip. Journal of Applied Mechanics 62, 362–367.